Perception Messages in Vehicular Networks: A Joint Analysis of Transmission Cost, Peak AoI, and Autocorrelation of Message Size

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***Abstract*—This paper addresses the optimization of the perception message periodicity in vehicular networks by minimizing a weighted sum of transmission and freshness costs. The expressions of the two contributions and the closed-form solution to the problem are obtained for the Dynamic Scheme of the New Radio Vehicle-to-Everything Side Link standard. Moreover, a model rooted in queueing theory is proposed to describe a connected automated vehicle as a source of messages about the detected objects. The model provides the statistical description of the number of objects in the vehicle viewing horizon, wherefrom the expression of its autocorrelation function is determined. Through the latter, the degree of similarity between the content of consecutive messages is understood. Lastly, an illustrative suburban scenario is investigated and realistically implemented in a simulator. The results show that increasing the CAV detection range is beneficial, allowing higher values of the optimal update periodicity, larger message sizes, and lower correlation levels between consecutive messages. This choice leads to obtaining the best communication performance, guaranteeing a higher probability of successful packet delivery.**

***Index Terms*—Connected Automated Vehicles, Cooperative Perception, NR-V2X SL, Peak Age of Information, Vehicular Networks**

I. Introduction

The perception messages broadcast by Connected Automated Vehicles (CAVs) will play a pivotal role in enhancing situational awareness and be decisive for the enablement of advanced Day 2 safety applications, such as vulnerable road user detection and occlusion handling, where onboard sensors alone are insufficient. The timeliness of the exchanged messages is crucial, and so is the transmission cost, as frequent updates cause high traffic levels and may result in congestion over the radio channel. One work investigating this aspect is [1], where the performance of several rate control algorithms for vehicular networks is evaluated in terms of congestion and Age of Information (AoI), although neglecting communication impairments. The authors of [2] focus on the AoI minimization problem in vehicular networks where cars employ 802.11p to communicate. The present work differs from the previous studies, as it introduces an analytical framework to identify the optimal periodicity of perception messages when connected vehicles directly communicate in accordance with the rules of the Dynamic Scheduling (DS) scheme of the New Radio - Vehicle-to-Everything Side Link (NR-V2X SL) standard. Namely, it puts forth and solves an optimization problem that jointly minimizes the transmission and freshness cost. Through a mathematical approach, the average number of objects the CAV perceives is determined, from which the transmission cost is assessed. To estimate the freshness cost, the average Peak AoI (PAoI) [3] the messages experience is obtained in closed form. Furthermore, a model based on queuing theory is introduced to identify the autocorrelation function (ACF) of the number of objects the CAV perceives at different times. The degree of similarity between the content of consecutive perception messages is then assessed. Some recent studies on this topic, such as [4] and [5], have analyzed the correlation between data transmitted by multiple sources to a common receiver through a game-theoretic approach. Unlike the former investigations, our work examines a real-world setting and focuses upon the CAV as a single message source.

The remainder of this paper is organized as follows. Section II formulates the optimization problem and derives the transmission and freshness cost. Section III identifies the optimal message periodicity. In Section IV, the CAV is modeled as a data traffic source and the ACF between the number of objects it perceives at different times is determined. Section V discusses the numerical results and Section VI draws the main conclusions.

II. Objective Function to Minimize

In the examined setting, the CAV detects nearby road occupants through its onboard sensors and notifies their presence to connected vehicles in the surroundings by periodically broadcasting perception messages. Our first goal is to find the best periodicity $T^{\star}_{up}$ to transmit the messages. To determine $T^{\star}_{up}$, an optimization problem is defined, where the objective function $C(T_{up})$ to minimize takes into account the cost of transmitting the messages, $C_{tr}(T_{up})$, and the freshness cost $C_{fr}(T_{up})$. Namely,

$$C(T_{up}) = (1-\alpha) \cdot \frac{C_{tr}(T_{up})}{\beta_{tr}} + \alpha \cdot \frac{C_{fr}(T_{up})}{\beta_{fr}} \qquad (1)$$

where: $\alpha \in [0,1]$ is the weighting factor, β_{tr} and β_{fr} are the normalization factors that ensure the two costs are comparable.

The transmission cost is defined as the average data rate the CAV produces, and as such, it is determined as the ratio between the average message size $\mathbb{E}[\mathcal{S}]$ and the period T_{up} at which the messages are generated, i.e.,

$$C_{tr}(T_{up}) = \frac{\mathbb{E}[\mathcal{S}]}{T_{up}} \,. \tag{2}$$

The freshness cost is defined as the average Peak Age of Information the packets experiment, that is,

$$C_{fr}(T_{up}) = \mathbb{E}[\text{PAoI}] \,. \tag{3}$$

In what follows $C_{tr}(T_{up})$ and $C_{fr}(T_{up})$ will be evaluated and the proper bounds to the optimization problem identified.

A. Transmission Cost

To determine the transmission cost borne by the CAV, it is observed that the message size is variable and can be expressed as

$$\mathcal{S} = S_0 + \mathcal{N} \cdot l_{bits} \tag{4}$$

where S_0 is a constant overhead, $\mathcal{N}$ is the number of perceived objects, and l_b is the number of bits carrying the information per single object.

To evaluate the average of $\mathcal{N}$, $\mathbb{E}[\mathcal{N}]$, it is assumed that the CAV travels along a road approximated by a straight line. Due to the onboard exteroceptive sensors, the CAV has a detection range D_R and experiences a viewing horizon $[x - D_R, x + D_R]$, where x denotes its position.

We make the hypothesis that the objects perceived by the CAV obey a Poisson distribution with an aggregate spatial rate $\hat{\Lambda}$, measured in objects per km. It follows that the average number of objects the CAV perceives is

$$\mathbb{E}[\mathcal{N}] = 2D_R \cdot \hat{\Lambda} \,. \tag{5}$$

Hence, from (2), (4), and (5), the transmission cost is obtained as:

$$C_{tr}(T_{up}) = \frac{S_0 + 2D_R \cdot \hat{\Lambda} \cdot l_b}{T_{up}} \,. \tag{6}$$

B. Radio Access Strategy and Freshness cost

To measure the recency of the received messages, we focus on the PAoI, which reflects how stale the most up-to-date information can be at its peak just before a new update message arrives. More accurately, we determine the average PAoI, $\mathbb{E}[\text{PAoI}]$, when the radio access strategy is the DS scheme of the NR-V2X SL standard. For this access solution, the CAV with a message ready for transmission at time t randomly and uniformly selects the required radio resources in a time window that opens at $t + T_1$ and ends at T_2, $T_{2,min} \leq T_2 \leq PDB$, where T_1 accounts for some processing time, $T_{2,min}$ depends on the OFDM subcarrier spacing, and PDB is the Packet Delay Budget, i.e., the maximum latency the message can tolerate [6]. In the following, $T_1 \simeq 0$ and $T_2 = PDB$, two widely accepted hypotheses in the literature [7], [8].

Assuming that the selection window is a temporal continuum, let U be a uniformly distributed random variable in $[0, T_2]$; then, denoting by t_i the time at which the CAV generates the i-th message and by t_{tx_i} the time at which the message is transmitted, we have

$$t_{tx_i} = t_i + U \,. \tag{7}$$

As the processing and propagation delays are negligible in single-hop short-range vehicular communications, the message is successfully received at time t_{rx_i}, well approximated by

$$t_{rx_i} = t_{tx_i} + t_s = t_i + U + t_s \tag{8}$$

where t_s denotes the OFDM slot duration.

Let K be the random variable that represents the number of consecutive message losses after the successful reception of the i-th message. If the next correctly received message is the $i+k+1$-th, the PAoI conditioned to $K = k$ losses is

$$\text{PAoI}_k = t_{rx_{i+k+1}} - t_i \,, \tag{9}$$

which making use of (8) becomes

$$\text{PAoI}_k = t_{i+k+1} + U + t_s - t_i \,. \tag{10}$$

The previous expression can be rewritten as:

$$\text{PAoI}_k = (t_{i+k+1} - t_{i+k}) + (t_{i+k} - t_{i+k-1}) + \ldots \\ \ldots + (t_{i+2} - t_{i+1}) + (t_{i+1} - t_i) + U + t_s \simeq \sum_{j=i}^{i+k} X_j + U, \tag{11}$$

where $X_j = t_{j+1} - t_j$ is the inter-generation time between the j-th and the $j+1$-th messages, and t_s contribution has been neglected, as $t_s << X_j, \forall j$. Recalling that the inter-generation time is constant and equal to the message periodicity T_{up}, $\mathbb{E}[\text{PAoI}]$ is obtained from (11) unconditioning with respect to k and U,

$$\mathbb{E}[\text{PAoI}] = (\mathbb{E}[K] + 1) \cdot T_{up} + \frac{T_2}{2} \,. \tag{12}$$

where $\mathbb{E}[K]$ denotes the average number of consecutive losses the message transmission incurs into. If messages are independently affected by losses and P_{succ} gives the probability of successful message reception, then K follows the geometric distribution with parameter P_{succ} and support $\{0, 1, 2, ...\}$. Accordingly,

$$\mathbb{E}[K] = \frac{1 - P_{succ}}{P_{succ}} \,. \tag{13}$$

Finally, replacing (13) in (12) and recalling that $T_2 = PDB$, we obtain the average PAoI, wherefrom

$$C_{fr}(T_{up}) = \mathbb{E}[\text{PAoI}] = \frac{T_{up}}{P_{succ}} + \frac{PDB}{2} \,. \tag{14}$$

III. Solution to the Optimization Problem

To solve the optimization problem previously formulated, the normalization factors in (1) have to be identified. We set β_{tr} to be the maximum value the transmission cost takes on; hence, recalling (6),

$$\beta_{tr} = \frac{S_{max}}{T_{min}}\,, \tag{15}$$

where S_{max} is the maximum message size and T_{min} the minimum update periodicity.

Analogously, β_{fr} is the maximum value assumed by the freshness cost in (14), wherefrom

$$\beta_{fr} = \frac{T_{max}}{P_{succ_{min}}} + \frac{PDB}{2}\,, \tag{16}$$

where T_{max} is the maximum update periodicity and $P_{succ_{min}}$ is the minimum acceptable P_{succ} value.

Recalling (1), (6), (14), (15), and (16), the optimization problem to solve is

$$\begin{aligned} &\min_{T_{up}} \left\{ \frac{(1-\alpha)}{\beta_{tr}} \cdot \frac{S_0 + 2D_R\hat{\Lambda}l_b}{T_{up}} + \frac{\alpha}{\beta_{fr}}\left(\frac{T_{up}}{P_{succ}} + \frac{PDB}{2}\right)\right\} \\ &\text{s.t.} \quad T_{min} \leq T_{up} < T_{max} \end{aligned} \tag{17}$$

where we set $T_{min} = T_2$, that is, $T_{up} \geq T_2 = PDB$, to ensure that messages are transmitted in the same order as they are generated.

To identify the optimal $T_{up}^{\star}$, we first consider the unbounded problem, whose solution is obtained after a few passages, also demonstrating that it is unique, as

$$\tilde{T}_{up} = \sqrt{\frac{(1-\alpha)\cdot(S_0 + 2D_R \cdot \hat{\Lambda} \cdot l_b)\cdot P_{succ} \cdot \beta_{fr}}{\alpha \cdot \beta_{tr}}}\,. \tag{18}$$

Recalling the bounds, the optimal update period is $T_{up}^{\star} = \tilde{T}_{up}$ if $PDB \leq \tilde{T}_{up} \leq T_{max}$; if $\tilde{T}_{up} < PDB$ or $\tilde{T}_{up} > T_{max}$, then $T_{up}^{\star} = PDB$ or $T_{up}^{\star} = T_{max}$ solve the bounded optimization problem, respectively.

IV. The CAV Model

Our next goal is to provide a thorough model of the vehicle as a traffic source. Observe that the CAV generates messages at a constant periodicity T_{up}; yet, (4) evidences that the message size is variable and linearly depends on the number of perceived objects $\mathcal{N}$, which varies over time. To statistically describe $\mathcal{N}$, assume that the objects perceived by the CAV belong to different classes (e.g., passenger cars, trucks, pedestrians) and that the distinctive features of every class are the velocity and the spatial rate of its perceived objects. Let the i-th class, $i = 1, 2, \ldots, H+1$, be characterized by a velocity v_i (where the v_i sign indicates whether objects travel in the same or opposite direction as the CAV), and spatial rate $\hat{\lambda}_i$. Given that p_i is the probability the object belongs to class i, then $\hat{\lambda}_i = p_i \cdot \hat{\Lambda}$. We let the class of objects moving at the same velocity as the CAV be the $H+1$-th. Denote the CAV velocity by v_{ego}; then observe that if an object belonging to class $H+1$ is "seen" by the CAV, it will indefinitely remain in its viewing horizon (unless at some point it will stop or change route).

It follows that $\mathcal{N}$ is the sum of two distinct contributions: the first is the number of perceived objects in class $H+1$, N_{H+1}, the second is the number of perceived objects belonging to the remaining classes. We therefore write the total number of objects in the CAV viewing horizon as

$$\mathcal{N} = N_{H+1} + N\,, \tag{19}$$

where N_{H+1} is Poisson-distributed.

To characterize $\mathcal{N}$, we model the CAV as a queuing system whose customers represent the objects of classes from 1 to H that enter the CAV viewing horizon. Our hypothesis is that the arrival process of the class i customers is Poisson with rate λ_i. If the CAV and the road objects travel in accordance to a linear uniform motion, then

$$\lambda_i = |v_{ego} - v_i| \cdot \hat{\lambda}_i \tag{20}$$

and the overall arrival rate is λ, $\lambda = \sum_{i=1}^{H} \lambda_i$. We assume that every object entering the CAV viewing horizon is perceived; it is a reasonable assumption, given the vehicle is equipped with video cameras, as well as lidars and radars. It therefore follows that the examined queuing system has an infinite number of servers.

Lastly, the service time d_i of a customer belonging to the i-th class coincides with the constant time interval the object of class i remains inside the CAV viewing horizon and is equal to

$$d_i = \frac{2 \cdot D_R}{|v_{ego} - v_i|}\,, \quad i = 1, 2, \ldots, H\,. \tag{21}$$

Hence, we have a queue, to which we refer by $M/mD/\infty$, where the service time is given by a mixture of deterministic distributions corresponding to H distinct customer classes (this explains the mD symbol). Accordingly, the average service rate is

$$\mu = \sum_{i=1}^{H} \frac{\lambda_i}{\lambda} \cdot d_i \tag{22}$$

The previous model allows us to conclude that N in (19) is the number of customers in the $M/mD/\infty$ queueing system. Hence, it is a random variable that obeys the Poisson distribution; its mean and variance, $\mathbb{E}[N]$ and $\mathbb{V}[N]$, are given by

$$\mathbb{E}[N] = \mathbb{V}[N] = \sum_{i=1}^{H} \lambda_i \cdot d_i \tag{23}$$

Next, the ACF of $\mathcal{N}$, denoted by $\tilde{\rho}(t)$, is determined. Its knowledge quantifies the degree of correlation between the number of objects the CAV perceives at different times, and ultimately reveals the correlation between the size of consecutive messages. Starting from (19), it can be demonstrated through standard methods that $\tilde{\rho}(t)$ is given by

$$\tilde{\rho}(t) = \rho(t) \cdot (1 - p_{H+1}) + p_{H+1}\,, \tag{24}$$

TABLE I: $T^\star_{up}$ variations in the P_{succ} range, $D_R = 50$ m.

	$T^\star_{up}$ [ms]		
$\hat{\Lambda}$ [obj/km]	50	100	200
$P_{succ} = 0.9$	139	192	268
$P_{succ} = 0.99999$	147	203	283

where $\rho(t)$ is the ACF of N and p_{H+1} is recalled to be the probability that an object belongs to class $H+1$. In the Appendix, it is proved that $\rho(t)$ is

$$\rho(t) = \begin{cases} 1 - \dfrac{t - \eta_i}{\mu}\,, & d_{i-1} \leq t < d_i \\ 0\,, & t \geq d_H \end{cases} \tag{25}$$

η_i being defined as $\eta_i = \sum_{k=1}^{i-1} p_k \cdot (t - d_k)$, with the service times ranked in increasing order, $d_1 < d_2 < \ldots < d_H$; for notational convenience $p_0 = 0$ and $d_0 = 0$.

V. Numerical Results

We analyze the solution to the optimization problem in a suburban scenario, populated by passenger cars and trucks, where the CAV velocity is $v_{ego} = 100$ km/h. The velocity of cars is ± 100 km/h, that of trucks is ± 70 km/h. Therefore, $H = 3$ customer classes participate in the queue. The fourth class includes the cars moving at the same CAV velocity. The message header size is $S_0 = 30$ bytes and the number of bits to describe a single object is $l_b = 456$. These values are drawn from the standard on the Cooperative Perception Messages (CPMs) [9]. Additionally, we select $S_{max} = 750$ bytes, considering the distributions in [10]. The minimum T_{up} periodicity corresponds to the lowest CPM inter-generation time, i.e., $T_{min} = 100$ ms [9]. We set $T_{max} = d_1$ to ensure that update messages include the fastest objects. Regarding the DS radio access strategy of NR-V2X SL, the selection window duration is set to $T_2 = 100$ ms to guarantee ordered packet reception, even in the limiting case where $T_{up} = T_{min}$. For safety applications (e.g., advanced driving, extended sensors), the success probability P_{succ} typically ranges from 0.9 to 0.99999 [6]. Henceforth, we set $P_{succ} = P_{succ_{min}} = 0.9$, corresponding to the minimum acceptable value.

Fig. 1 shows the optimal periodicity $T^\star_{up}$ as a function of the detection range D_R for three aggregate spatial rates, $\hat{\Lambda} = 50, 100, 200$ obj/km; the weighting factor is set to $\alpha = 0.8$ to prioritize message freshness over the transmission cost. As revealed by (18), for a given $\hat{\Lambda}$, $T^\star_{up}$ increases with D_R: objects remain within the viewing horizon for a longer period, allowing messages to be transmitted less frequently. This helps reduce channel load while preserving adequate information freshness. Furthermore, higher values of $T^\star_{up}$ are observed as $\hat{\Lambda}$ increases: a higher $\hat{\Lambda}$ value implies a higher average transmission cost, which can be mitigated by adopting a longer update interval. Interestingly, Table I shows that, when freshness is prioritized over transmission cost ($\alpha = 0.8$) $T^\star_{up}$ varies only slightly as P_{succ} increases from 0.9 to 0.99999, regardless of the $\hat{\Lambda}$ value.

Fig. 2 shows $\tilde{\rho}(t)$, the ACF of the number of perceived objects, $\mathcal{N}$, for three different combinations of the $[p_1, p_2, p_3, p_4]$ probabilities. The rationale is to reflect either a clear prevalence of cars over trucks, or vice versa, or a fair split. Two reference $T^\star_{up}$ values are highlighted in the figure: (i) $T^\star_{up1} = 139$ ms, the optimal periodicity when $D_R = 50$ m, $\hat{\Lambda} = 50$ obj/km, and $\alpha = 0.8$; (ii) $T^\star_{up2} = 268$ ms, obtained when $D_R = 50$ m, $\hat{\Lambda} = 200$ obj/km, and $\alpha = 0.8$. An increase in $\hat{\Lambda}$, and thus a higher optimal periodicity, results in lower ACF values. Additionally, we can observe that the occurrence probabilities influence the ACF shape, as they determine the number of objects across different categories, each with distinct relative speeds and service times.

Finally, to evaluate the communication performance, we recreated the road setting using the widely tested MoReV2X simulator [11] [12]. The main system choices and simulation parameters are reported in Table II. Connected vehicles generate periodic traffic with period $T^\star_{up}$ equal to 139, 270, and 662 ms, corresponding to $\hat{\Lambda} = 50$ vehicles/km and $D_R = 50$, 100, and 250 m. According to (4) and the model outcomes, the message size is given by a constant contribution (due to the overhead S_0 and the average number of objects having the same CAV velocity) plus a term that follows a truncated Poisson distribution. The truncation ensures that the maximum allowed message size is not exceeded, given the selected Modulation and Coding Scheme (MCS) and the available number of subchannels. Fig.3 depicts the obtained P_{succ} as a function of $d_{t,r}$, the distance between the transmitting CAV and the vehicle receiving the messages. A key observation is that increasing D_R results in larger packets being transmitted

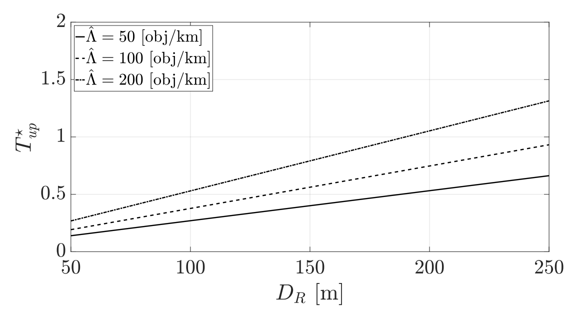

Fig. 1: $T^\star_{up}$ as a function of D_R, $\alpha = 0.8$.

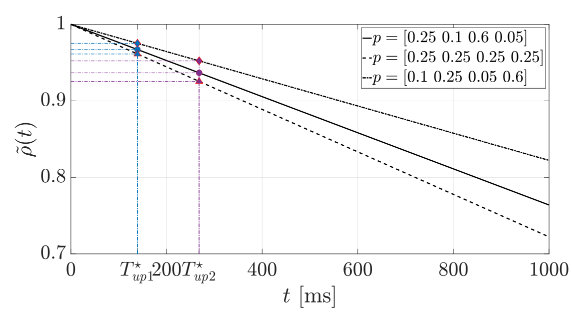

Fig. 2: ACF of the number of perceived objects.

TABLE II: Simulation choices and parameters.

System choices	
Scheduling scheme	DS
Selective fading	clustered delay lines
Shadowing	lognormal
Parameter	**Value**
Center frequency	5.9 GHz
Channel bandwidth	20 MHz
SCS	30 KHz
Modulation type	16 QAM
Total number of subchannels	4
MCS	13
Transmission power	23 dBm
Receiver sensitivity level	−103.5 dBm
Noise figure	9 dBm

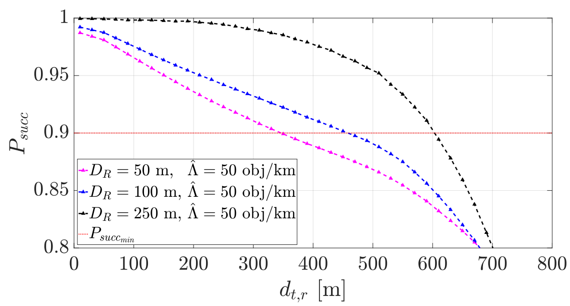

Fig. 3: P_{succ} for $\hat{\Lambda} = 50$ objects/Km.

less frequently. This is shown to always guarantee higher P_{succ} values; as a consequence, the distance at which $P_{succ} = 0.9$ significantly increases (it is 600 m when $D_R = 250$ m).

VI. Conclusions

This paper set forth an optimization problem to determine the most suitable perception message periodicity by minimizing the weighted sum of transmission and freshness costs in a vehicular network adopting the 5G NR-V2X SL technology. The expressions for both costs have been derived, and the closed-form solution to the problem has been obtained. Additionally, a queueing theory-based model has been developed to statistically characterize the number of objects perceived by a connected vehicle and its autocorrelation function. The latter has provided useful insights on the degree of similarity between consecutive messages. Finally, an illustrative suburban scenario has been analyzed through realistic simulations. It has been found that it is worth increasing the CAV detection range, as this implies higher values of the optimal periodicity paired with larger message sizes and lower correlation between consecutive messages. This choice was also demonstrated to achieve the best communication performance, warranting a higher probability of successful packet delivery.

Appendix

We prove that $\rho(t)$ obeys the expression in (25). The ACF of N is defined as

$$\rho(t) = \frac{\mathbb{E}[N(t)N(0)] - \mathbb{E}[N]^2}{\mathbb{V}[N]}\,, \qquad (26)$$

where $\mathbb{E}[N]$ and $\mathbb{V}[N]$ are given in (23) and

$$\mathbb{E}[N(t)N(0)] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} mn\cdot\frac{(\mathbb{E}[N])^m}{m!}e^{-\mathbb{E}[N]}\cdot p_{mn}(t)\,, \qquad (27)$$

with $p_{mn}(t)$, $m = 0, 1, \ldots$, $n = 0, 1, \ldots$, being the transient probability of moving from state $N = m$ at time 0 to state $N = n$ at time t. Specializing the findings in [13], the $p_{mn}(t)$ expression is obtained as

$$p_{mn}(t) = \qquad (28)$$
$$\begin{cases} \sum_{r=0}^{\min(n,m)} \binom{m}{r} \frac{(1-\gamma_i)^r \gamma_i^{m-r} (\lambda\psi_i)^{n-r} e^{-\lambda\psi_i}}{(n-r)!}, & d_{i-1} \le t < d_i \\ \frac{(\lambda\mu)^n}{n!} e^{-\lambda\mu}, & t \ge d_H \end{cases}$$

where $\psi_i = t - \eta_i$, $\gamma_i = \psi_i/\mu$, $i = 1, 2, \ldots, H$, and $\eta_i = \sum_{k=1}^{i-1} p_k \cdot (t - d_k)$. Briefly, in the first branch r is used as the number of customers still in the system out of the m initially present customers, so we need $n - r$ arrivals to have n customers at time t, and we must have $r \le m$ and $r \le n$. For both the remaining and arriving customers, the service time distribution is used to calculate the probability that a customer is present at time t. In the second branch, all customers present initially leave the system up to time t and the transient probabilities are equal to the steady-state probabilities. In the interval $d_{i-1} \le t < d_i$, $i = 1, 2, \ldots, H$, $p_{mn}(t)$ is more aptly rewritten as

$$\begin{aligned} p_{mn}(t) &= \\ &= \sum_{r=0}^{min(n,m)} \binom{m}{r} \left(\frac{\mu-\psi_i}{\mu}\right)^r \left(\frac{\psi_i}{\mu}\right)^{m-r} \frac{(\lambda\psi_i)^{n-r}}{(n-r)!} e^{-\lambda\psi_i} = \\ &= \sum_{r=0}^{m} \binom{m}{r} \left(\frac{\psi_i}{\mu}\right)^m \left(\frac{\mu-\psi_i}{\lambda\psi_i^2}\right)^r \frac{(\lambda\psi_i)^n e^{-\lambda\psi_i}}{(n-r)!}\,, \end{aligned} \qquad (29)$$

where the upper limit of the summation becomes m because $1/(n-r)! = 0$ for $n - r < 0$. Replacing (29) in (27), we have

$$\begin{aligned} \mathbb{E}[N(t)N(0)] &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} mn \frac{(\lambda\mu)^m e^{-\lambda\mu}}{m!}\cdot \\ &\cdot \sum_{r=0}^{m} \binom{m}{r} \left(\frac{\psi_i}{\mu}\right)^m \left(\frac{\mu-\psi_i}{\lambda\psi_i^2}\right)^r \frac{(\lambda\psi_i)^n e^{-\lambda\psi_i}}{(n-r)!} = \\ &= \sum_{m=0}^{\infty}\sum_{r=0}^{m} \frac{\lambda^m e^{-\lambda(\mu+\psi_i)}}{(m-1)!} \binom{m}{r}\cdot \\ &\cdot \psi_i^{m-r} (\mu-\psi_i)^r \sum_{n=0}^{\infty} \frac{n(\lambda\psi_i)^{n-r}}{(n-r)!}\,. \end{aligned} \qquad (30)$$

In (30), the sum in n can be rewritten as

$$\sum_{n=0}^{\infty} \frac{n(\lambda\psi_i)^{n-r}}{(n-r)!} = \\ = \sum_{n=0}^{\infty} (n-r)\frac{(\lambda\psi_i)^{n-r}}{(n-r)!} + \sum_{n=0}^{\infty} \frac{r(\lambda\psi_i)^{n-r}}{(n-r)!} \ , \quad (31)$$

where the first term converges to $(\lambda\psi_i)e^{\lambda\psi_i}$ and the second to $re^{\lambda\psi_i}$. Therefore, the sum converges to $e^{\lambda\psi_i}(r + \lambda\psi_i)$. Replacing (31) into (30) yields

$$\mathbb{E}[N(t)N(0)] = \\ = \sum_{m=0}^{\infty}\sum_{m=0}^{r} \frac{\lambda^m e^{-\lambda\mu}}{(m-1)!}\binom{m}{r}\psi_i^{m-r}(\mu-\psi_i)^r(r+\lambda\psi_i) = \\ = \sum_{m=0}^{\infty} \frac{\lambda^m e^{-\lambda\mu}}{(m-1)!}\left(\sum_{r=0}^{m}\binom{m}{r}\psi_i^{m-r}(\mu-\psi_i)^r r + \right. \\ \left. + \lambda\psi_i \sum_{r=0}^{m}\binom{m}{r}\psi_i^{m-r}(\mu-\psi_i)^r\right) . \quad (32)$$

The two inner sums are binomial sums. The first can be equivalently formulated as

$$\sum_{r=0}^{m}\binom{m}{r}\psi_i^{m-r}(\mu-\psi_i)^r r = \\ = \sum_{r=0}^{m-1}\binom{m-1}{r-1} m\psi_i^{m-r}(\mu-\psi_i)^{r-1}(\mu-\psi_i) = \\ = \mu^{m-1}m(\mu-\psi_i) , \quad (33)$$

while the second is already in canonical form and converges to μ^m. Hence, substituting in (33),

$$\mathbb{E}[N(t)N(0)] = \\ = \sum_{m=0}^{\infty} \frac{\lambda^m e^{-\lambda\mu}}{(m-1)!}(\mu^{m-1}m(\mu-\psi_i) + \lambda\psi_i\mu^m) = \\ = e^{\lambda\mu}\left((\mu-\psi_i)\lambda\sum_{m=0}^{\infty}\frac{(\lambda\mu)^{m-1}m}{(m-1)!} + \right. \\ \left. + \lambda^2\psi_i\mu\sum_{m=0}^{\infty}\frac{(\lambda\mu)^{m-1}}{(m-1)!}\right) = \\ = e^{-\lambda\mu}\left((\mu-\psi_i)\lambda e^{\lambda\mu}(1+\lambda\mu) + \lambda^2\psi_i\mu e^{\lambda\mu}\right) = \\ = \lambda(\mu-\psi_i+\lambda\mu^2) . \quad (34)$$

Finally, replacing (23) and (34) in (26), the ACF becomes

$$\rho(t) = \frac{\lambda(\mu-\psi_i+\lambda\mu^2)-(\lambda\mu)^2}{\lambda\mu} = \\ = 1 - \frac{\psi_i}{\mu} = 1 - \frac{t-\eta_i}{\mu} , \quad d_{i-1} \le t < d_i \quad (35)$$

For $t > d_H$, where d_H is the longest service time, all clients present at time 0 already left the system, which is now in the stationary regime; therefore, there is no correlation between $N(0)$ and $N(t)$, i.e., $\rho(t) = 0$. This is formally demonstrated observing that the transient probability, $p_{mn}(t)$, is now the stationary probability of a Poisson process, given by the second branch of (28), which replaced in (27) gives

$$\mathbb{E}[N(t)N(0)] = \\ = \sum_{m=0}^{\infty} m\frac{(\lambda\mu)^m}{m!}e^{-\lambda\mu}\sum_{n=0}^{\infty} n\frac{(\lambda\mu)^n}{n!}e^{-\lambda\mu} = (\lambda\mu)^2 \quad (36)$$

wherefrom, recalling (26),

$$\rho(t) = \frac{(\lambda\mu)^2-(\lambda\mu)^2}{\lambda\mu} = 0 . \quad (37)$$

Combining (35) and (37), (25) is finally obtained. □

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