# A Novel Queuing Theory Approach for Modeling Connected Automated Vehicles as Traffic Sources


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***Abstract*—In the near future, Cooperative Perception Services and Cooperative Perception Messages will be the main ingredients for the achievement of high levels of road safety. Connected Automated Vehicles are expected to be an integral part of this scenario, as they gather information through a multitude of sensors and share it with other vehicles and road infrastructures via vehicular communications. This in-progress work proposes a novel analytical model to describe what type of traffic a Connected Automated Vehicle would generate in a simplified driving scenario. The model is based on queueing theory and considers the vehicle as the queuing system, while the perceived objects are the customers of the queue. Some preliminary results are derived, demonstrating that there is a linear dependency between the average size of the vehicle's messages and the vehicle's speed and density of the objects. Moreover, the autocorrelation function of the number of objects the vehicle perceives rapidly decays and reaches the zero in a time which depends on the vehicular speed and the Field of View of the vehicle's sensors.**



***Index Terms*—Road safety, Cooperative Perception Messages, Connected Automated Vehicles, Queueing Theory, Traffic Modeling**


## I. Introduction

Connected Automated Vehicles (CAVs) are promising elements in the framework of Intelligent Transport Systems (ITS). They reap information about the surrounding environment, owing to the variety of onboard sensors (cameras, RADARs, LiDARs, IMU, and GPS) they are equipped with. As safety applications are progressing from Day-1 to Day-2, such information becomes crucial for successfully implementing Cooperative Perception Services (CPSs). CPSs allow information sharing among vehicles, Road-Side Units (RSUs), and other road occupants to increase road safety, relieve congestion, and contribute to a more sustainable usage of the road infrastructure. The European Telecommunications Standard Institute (ETSI) indicates that the information exchanged by CAVs should be organized in the form of Cooperative Perception Messages (CPMs), as detailed in the recently released standard TS 103-324 [1]. These messages carry useful information about the objects perceived by the vehicle and are disseminated with a periodicity whose value lies in the [100 ms, 1 s] range. CPMs are composed of several containers, the most relevant being the Perceived Object Container (POC), where a list of attributes of the objects the CAV detects is recorded. A perceived object is reported in the POC only if some conditions about its position, speed, and heading are satisfied. From the CPM format, it is clear that the number of perceived objects reported in the POC determines the CPM size for the most part, as well as the correlation among the size of consecutive messages.

The authors in [2] analyze the CPM generation rules along with their impact on the communication channel. Various additional works concentrate on the analysis of CPMs generated by cooperating vehicles and propose redundancy mitigation rules to decrease the channel load [2], [3], [4]. However, none of the above works engages in the analytical study of CAV traffic. This work puts forth a model based on queueing theory, to obtain a statistical characterization of the number of objects each CPM advertises. The aim is to gain a mathematical understanding of the CPM traffic the CAV pours on the radio channel; upcoming studies may leverage it to envisage CPM mitigation techniques and strategies to optimally fuse the information collected and shared by different vehicles. The main outcomes of the present study are:

- the derivation of the model equations;
- the characterization of the average number of objects perceived and included in a CPM and the autocorrelation function of such number;
- the proof that the average number of detected objects linearly increases with the FoV of the vehicle's sensors and linearly decreases with the vehicle's speed.

Accordingly, the paper is organized as follows: in Section II, the considered model, its underlying assumptions, and some potential model enhancements are discussed; In Section III, a few preliminary results are provided, while Section IV draws some first conclusions.

## II. System Model

Traditionally, the studies applying queueing theory to the vehicular environment focus on road traffic; they assume that vehicles are the customers, while the road lanes constitute the queueing systems [5], [6], [7], [8]. The proposed approach is


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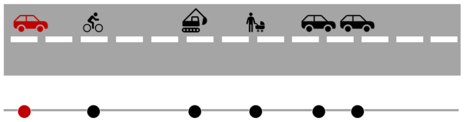

Fig. 1. Road model.

different, as the goal is to characterize the CPM data traffic, deriving useful information about the messages the CAV generates. Recalling that the CPM size is primarily determined by the objects the CAV detects, the focus is therefore placed on these objects, which become the customers, while the CAV is the queuing system of the proposed model. Moreover, the time interval during which the CAV's onboard sensors detect an object is the customer's service time. Assuming that all the objects which fall in the vehicle's viewing horizon are detected, the number of servers is infinite and there is no notion of waiting time. The missing and more challenging element is the characterization of the arrival and service processes.

### A. The Simplest Scenario

To identify the way customers arrive at the system and for how long they have to be served, a very simple scenario is examined first. To describe it, the following hypotheses are introduced:

i. the road is approximated as a straight line, i.e., a one-dimensional representation is adopted, as portrayed in Fig.1;
ii. the CAV travels following a uniform rectilinear motion with speed $|v|$ km/h;
iii. the objects along the road (e.g., other vehicles, bicycles, pedestrians) are not moving;
iv. the objects' location is modeled through a Poisson Point Process (PPP) having rate $\lambda_s$ objects/km;
v. for a given Field of View (FoV) (in m) guaranteed by the onboard sensors, the CAV located at position $x$ has a viewing horizon spanning in the range $x - \text{FoV}$ to $x + \text{FoV}$.

Owing to assumptions *ii.* and *iv.*, the arrival process to the queuing system is Poisson; owing to *ii.* and *iii.*, the service time is deterministic. Therefore, we can model this scenario as a $M/D/\infty$ queueing system. The rate at which customers enter the system is denoted by $\lambda$ and is given by

$$\lambda = \frac{\lambda_s \cdot |v|}{3.6 \cdot 10^3} \quad \text{objects/s}\,. \tag{1}$$

As regards the service time, its expression is derived recalling that the CAV has a uniform rectilinear motion. If at time $t = 0$ its position along the road is $x_0 = 0$, an object located at $\tilde{x}$ enters the vehicle's viewing horizon at $t^{'}$,

$$t' = \frac{\tilde{x} - \text{FoV}}{|v|} \tag{2}$$

and leaves it at $t''$,

$$t'' = \frac{\tilde{x} + \text{FoV}}{|v|}\,; \tag{3}$$

the service time $d$ is then computed as the difference between these two instants:

$$d = t'' - t' = \frac{2\text{FoV}}{|v|}\ \text{s}\,. \tag{4}$$

Our primary focus revolves around statistically characterizing the quantity of customers present within the system, denoted as $N$. Indeed, the number of objects within the CAV's viewing horizon is the determining factor that governs the actual size of the CPM payload.

For the $M/D/\infty$ system operating in steady state, the probability of $m$ customers in system, $p_m = P[N = m]$, can be written as:

$$p_m = \frac{(\lambda d)^m}{m!} e^{-\lambda d}\,, \quad m = 0, 1, \dots\ , \tag{5}$$

i.e., $N$ obeys a Poisson distribution [9], wherefrom the mean and the variance of the number of customers in the system, $\mathbb{E}[N]$ and $\mathbb{V}[N]$, respectively, are immediately obtained as

$$\mathbb{E}[N] = \mathbb{V}[N] = \lambda d\,. \tag{6}$$

It is also convenient to evaluate the autocorrelation function of the number of customers, denoted by $c(\tau)$, given as

$$c(\tau) = \frac{\mathbb{E}[N(\tau)N(0)] - \mathbb{E}[N]^2}{\mathbb{V}(N)}\,, \quad \tau \geq 0\,. \tag{7}$$

where $\mathbb{E}[N(\tau)N(0)]$ can be calculated as

$$\mathbb{E}[N(\tau)N(0)] = \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} m \cdot n \cdot p_m \cdot p_{mn}(\tau)\,. \tag{8}$$

with $p_{mn}(t)$, $m = 0, 1 \dots$ and $n = 0, 1, \dots$ being the transient probability of moving from state $m$ at time 0 to state $n$ at time $t$ of the M/D/$\infty$ queue in steady-state.

By leveraging the findings in [10] it can be shown that

$$p_{mn}(t) = \begin{cases} \displaystyle\sum_{r=0}^{min(n,m)} \binom{m}{r} \alpha \frac{(\lambda t)^{n-r}}{(n-r)!} e^{-\lambda t} & t \leq d \\ \frac{(\lambda d)^n}{n!} e^{-\lambda d} & t > d \end{cases} \tag{9}$$

where $\alpha = \left(\frac{d-t}{d}\right)^r \left(\frac{t}{d}\right)^{m-r}$. Eq. (9) can be explained as follows. In case $t \leq d$, the number of customers in the queue is the sum of the number of customers that were initially present and are still present (denoted by $r$) and the number of those that arrived in the interval $[0, t]$ (in order to have $n$ customers at time $t$ their number must be $n - r$). Since in steady state the remaining service time of a customer is uniform on $[0, d]$, the probability that an initially present customer is still in the system at time $t$ is $(d - t)/d$, that is, it is the probability that its residual service time exceeds $t$. On the contrary, the probability that a given customer initially present is not in the system at time $t$ is $t/d$, as its residual service time has to be lower than $t$. The $r$ customers still in the queue can be chosen in $\binom{m}{r}$ ways and their number lies between 0 and $min(n, m)$. On the other hand, $n - r$, the number of customers that arrive in $[0, t]$, follows the Poisson distribution with parameter $\lambda t$, wherefrom the first branch of (9) is obtained. In case $t > d$, the initially present $m$ customers are not in the queue any longer and the number of customers in the queue is equal to the number of customers that arrived in $[t-d, t]$. As the number of customers that arrive in an interval of length $d$ is Poisson distributed with parameter $\lambda d$, the second branch of (9) follows.

### B. Future Work: More Realistic Settings

From this simple scenario, it is possible to move further in different directions. To reflect the presence of various categories of non-stationary objects, that is, road users such as pedestrians, bicycles, and vehicles moving at different speeds, distinct classes of customers can be introduced, each class with a unique service time. Assuming $M$ classes of objects and the object's speed of the $i$-th class being $|v_i|$, the service time of the $i$-th class is:

$$d_i = \frac{2\text{FoV}}{|v| \pm |v_i|} \tag{10}$$

where the minus sign corresponds to the objects moving in the same direction as the CAV and the plus sign to the objects moving in the opposite direction. Moreover, to render different levels of traffic congestion that the CAV may encounter along the road, a non-homogeneous arrival rate $\lambda(t)$ can be introduced.

## III. Preliminary Results

The first interesting parameter to visualize is $\mathbb{E}[N]$, the average number of objects the CAV perceives. Fig.2 displays $\mathbb{E}[N]$ as a function of the arrival rate $\lambda$, when FoV $= 50$ m, a reasonable value for commercial automotive RADARs or LiDARs, and for three distinct $|v|$ values, namely, $|v| = 50$, 100, and 130 Km/h. As expected, $\mathbb{E}[N]$ increases as the arrival rate $\lambda$ increases and decreases as the CAV speed increases. Last trend is because a higher CAV speed reflects in a lower service time, as the objects more rapidly enter and leave the sensors' FoV. The autocorrelation function $c(\tau)$ is reported in Fig.3, for FoV $= 50$ m and $|v| = 50$, 100 and 130 km/h, having set the arrival rate to $\lambda = 1$ obj/s. It can be noticed that $c(\tau)$ always starts from 1, i.e., from a situation of completely positive correlation, and linearly decreases to zero when $\tau = d$, which takes on the value of 7.2, 3.6 and 2.8 seconds in the three cases under analysis. The linear behavior is due to the deterministic service time. Additionally, $c(\tau)$ is independent of the arrival rate $\lambda$. These claims will be demonstrated in future work.

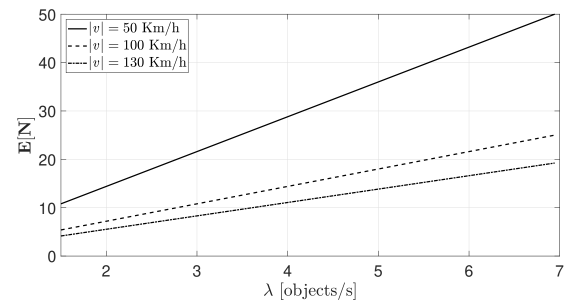


Fig. 2. $\mathbb{E}[N]$ as a function of $\lambda$.

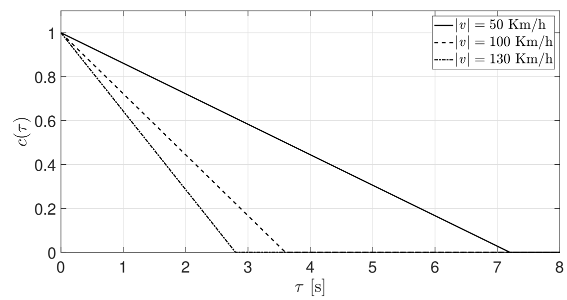


Fig. 3. $c(\tau)$, autocorrelation function of $N$.

## IV. Conclusions and Forthcoming Work

This paper has proposed a simple mathematical model to capture the most salient characteristics of the CPMs broadcasted by CAVs on the radio channel. The model allows to statistically characterize the number of objects detected by the vehicle, as it is this number that determines the CPM size. Under the simplifying assumptions which have been adopted, the work demonstrates that the average CPM size linearly increases with the density of the objects the CAV detects, and linearly decreases with its speed. Moreover, the autocorrelation function of the number of objects the CAV perceives goes to zero in a time which is the interval during which the objects are "seen" by the CAV. As this work is still in progress, the model needs to be improved by removing some initial, simplifying assumptions. Furthermore, the obtained results have to be validated via real-world data and realistic simulations.

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